B.Sc., MATHEMATICS/ BCA/B.SC COMPUTER SCIENCE/CAMA15B

MATHEMATICAL FOUNDATION - I

E CONTENT

OBJECTIVES

To know about Logical operators, validity of arguments, set theory and set Operations, relations and functions

UNIT: SYMBOLIC LOGIC

Proposition, Logical operators, conjunction, disjunction, negation, conditional and bi conditional operators, converse, Inverse, Contra Positive, logically equivalent, Tautology and contradiction. Arguments and validity of arguments.

UNIT: SET THEORY

Sets, set operations, Venn diagram, Properties of sets, number of elements in a Set, Cartesian product, relations & functions, Relations: Equivalence relation. Equivalence class, partially and totally Ordered sets, Functions: Types of Functions, Composition of Functions.

UNIT: SYMBOLIC LOGIC

Proposition:

- A proposition is a statement which can be classified as true or false. The true or false of a proposition is called truth value of a proposition. These two values true and false are denoted by the symbols *T* and *F* respectively.

The following statements are propositions.

1. Chennai is the capital of Tamilnadu. - True
2. 2 + 3=6 - False
3. 5 is a prime number – True

The following are not propositions.

1. What are you doing?
2. Take one book.
3. x + y = z.

Logical operations:

- There are several ways in which we commonly combine simple propositions into compound ones. In order to produce compound propositions from simple ones we use words and, or, not, if.

Operators	**Symbol**	Word
Conjunction	Λ	And
Disjunction	V	Or
Negation	~	Not
Conditional	→	If
Biconditional	↔	If and only if

Conjunction:

- Let p and q be two propositions. the proposition "p and q", denoted by p Λ q
- p Λ q is true when both p and q are true and is false otherwise.
- The proposition p Λ q is called conjunction of p and q
- Truth Table

P	q	p Λ q
T	T	T
T	F	F

F	T	F
F	F	F

Disjunction:

- Let p and q be two propositions. the proposition “p or q”, denoted by p v q
- p v q is false when both p and q are false and is true otherwise.
- The proposition p v q is called disjunction of p or q
- Truth Table

P	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

Negation:

- Let p be the propositions. It is not the case that p is another proposition, called the negation of p.
- The negation of p is denoted by $\sim p$ (or)$\neg p$
- The proposition $\sim p$ is read as “not p”
- Truth table

P	$\sim p$
T	F
F	T

Conditional:

- Let p and q be two propositions when both p and q are false and is true otherwise.
- The implication “p → q” is the proposition p v q is false when p is true and q is false and true otherwise .
- In this p is called the hypothesis (or antecedent or premise) and q is called the conclusion (or consequence)
- Truth Table

P	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional:

- Let p and q be two propositions
- The biconditional $p \leftrightarrow q$ is the proposition that it is true when p and q have the same

truth values and is false otherwise

- Truth Table

P	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Converse:

- Let p and q be two propositions. p → q is a conditional proposition.
- The proposition q → p is called converse of the proposition p → q .

P	q	p → q	q → p
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Inverse:

- Let p and q be two propositions. The proposition ~p → ~q is called inverse of the proposition

p	q	~p	~q	~p → ~q
T	T	F	F	T
T	F	F	T	T

F	T	T	F	F
F	F	T	T	T

Contrapositive:

- Let p and q be two propositions. The proposition ~q → ~p is called contrapositive of the proposition

P	q	~p	~q	~q → ~p
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Tautology, Contradiction and Contingency:

- A compound proposition that is always true, no matter what the truth values of the propositions that occur is called a tautology.
- A compound proposition that is always false is called contradiction.
- A proposition that is neither tautology nor a contradiction is called Contingency.

Example for tautology:

P	~p	P v ~ p
T	*F*	*T*
F	*T*	*T*

Since the last column of P v ~ p contains only T, P v ~ p is a tautology

Example for contradiction:

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

Since the last column contains only F, $p \wedge \neg p$ is a contradiction.

Logically equivalent:

- The propositions p and q are called logically equivalent if p $\leftrightarrow$ q is a tatulogy. The notation $\Leftrightarrow$ is used to denote p and q are logically equivalent.
- It is denoted by "≡".

Example for logically equivalent:

Show that ~ (p Λ q) ≡ ~pv~q are logically equivalent.

P	q	~p	~q	p Λ q	~ (p Λ q)	~pv~q
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

From the last two columns ~ (p Λ q) ≡ ~pv~q are logically equivalent.

Laws of algebra of propositions:

Some standard equivalent statements:

a. Idempotent Law:

i. $p \vee p \equiv p$

ii. $p \wedge p \equiv p$

b. Commutative Law:

i. $p \vee q \equiv q \vee p$

ii. $p \wedge q \equiv q \wedge p$

c. Associative Law:

i. $(p \vee q) \vee r \equiv p \vee (q \vee r) \equiv p \vee q \vee r$

ii. (p ∧ q) ∧ r ≡ p ∧ (q ∧ r) ≡ p ∧ q ∧ r

d. Distributive Law:

i. p ∨ (q ∧ r) ≡ (p ∨ q) ∧ (p ∨ r)

ii. p ∧ (q ∨ r) ≡ (p ∧ q) ∨ (p ∧ r)

e. Identity Law:

i. p ∨ F ≡ p

ii. p ∧ F ≡ F

iii. p ∨ T ≡ T

iv. p ∧ T ≡ p

f. Complement Law:

i. p ∨ ~p ≡ T

ii. p ∧ ~p ≡ F

iii.

g. Involution Law:

i. ~T ≡ F

ii. ~F ≡ T

iii. ~(~p) ≡ p

h. DeMorgan's Law:

i. ~(p ∨ q) ≡ ~p ∧ ~q

ii. ~(p ∧ q) ≡ ~p ∨ ~q

Proof for Associative Laws:

i. (p ∨ q) ∨ r ≡ p ∨ (q ∨ r)

ii. (P ∧ q) ∧ r ≡ P ∧ (q ∧ r)

The truth table required for proving the associative law is given below.

p	*q*	*r*	*p* ∨ *q*	*q* ∨ *r*	(p ∨ q) ∨ r	p ∨ (q ∨ r)
T	*T*	*T*	*T*	*T*	*T*	*T*
T	*T*	*F*	*T*	*T*	*T*	*T*
T	*F*	*T*	*T*	*T*	*T*	*T*
T	*F*	*F*	*T*	*F*	*T*	*T*
F	*T*	*T*	*T*	*T*	*T*	*T*
F	*T*	*F*	*T*	*T*	*T*	*T*
F	*F*	*T*	*F*	*T*	*T*	*T*
F	*F*	*F*	*F*	*F*	*F*	*F*

The columns corresponding to (p ∨ q) ∨ r and p ∨ (q ∨ r) are identical.

Hence $(p \vee q) \vee r \equiv p \vee (q \vee r)$

Similarly, (ii) $(P \wedge q) \wedge r \equiv P \wedge (q \wedge r)$ can be proved.

Proof for Distributive laws:

i. $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ ii. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

p	q	r	$(q \wedge r)$	$P \vee (q \wedge r)$	$(p \vee q)$	$(p \vee r)$	$(p \vee q) \wedge (p \vee r)$
T	*T*	*T*	*T*	*T*	*T*	*T*	*T*
T	*T*	*F*	*F*	*T*	*T*	*T*	*T*
T	*F*	*T*	*F*	*T*	*T*	*T*	*T*
T	*F*	*F*	*F*	*T*	*T*	*T*	*T*
F	*T*	*T*	*T*	*T*	*T*	*T*	*T*
F	*T*	*F*	*F*	*F*	*T*	*F*	*F*
F	*F*	*T*	*F*	*F*	*F*	*T*	*F*
F	*F*	*F*	*F*	*F*	*F*	*F*	*F*

The columns corresponding to $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are identical.

Hence $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.

Similarly (ii) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ can be proved.

Proof for De Morgan's Laws:

i. $\neg (p \wedge q) \equiv \neg p \vee \neg q$. ii. $\neg (p \vee q) \equiv \neg p \wedge \neg q$

p	***q***	***¬p***	***¬q***	$p \wedge q$	$\neg (p \wedge q)$	$\neg p \vee \neg q$.
T	*T*	*F*	*F*	*T*	*F*	*F*
T	*F*	*F*	*T*	*F*	*T*	*T*
F	*T*	*T*	*F*	*F*	*T*	*T*
F	*F*	*T*	*T*	*F*	*T*	*T*

The entries in the columns corresponding to $\neg (p \wedge q)$ and $\neg p \vee \neg q$.

are identical and hence they are equivalent. Therefore . $\neg (p \wedge q) \equiv \neg p \vee \neg q$. Dually (ii) $\neg (p \vee q) \equiv \neg p \wedge \neg q$ can be proved.

Example :1

Prepare the truth table of the followingstatement patterns:

i. $[(p \to q) \wedge q] \to p$

ii. $(p \wedge q) \to (\sim p)$

iii. $(p \leftrightarrow r) \wedge (q \leftrightarrow p)$

iv. $(p \vee \sim q) \to (r \wedge p)$

Solution:

i. $[(p \to q) \wedge q] \to p$

P	q	$p \to q$	$(p \to q) \wedge q$	$[(p \to q) \wedge q] \to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

ii. $(p \wedge q) \to (\sim p)$

P	q	$p \wedge q$	$\sim p$	$(p \wedge q) \to (\sim p)$
T	T	T	F	F
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

iii. $(p \leftrightarrow r) \wedge (q \leftrightarrow p)$

p	q	r	$p \leftrightarrow r$	$q \leftrightarrow p$	$(p \leftrightarrow r) \wedge (q \leftrightarrow p)$
T	T	T	T	T	T
T	T	F	F	T	F
T	F	T	T	F	F
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	T	F	F
F	F	T	F	T	F
F	F	F	T	T	T

iv. $(p \vee \sim q) \to (r \wedge p)$

P	q	r	~q	p v ~ q	r ∧ p	(p v ~q) → (r ∧ p)
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	F	T
F	T	F	F	F	F	T
F	F	T	T	T	F	F
F	F	F	T	T	F	F

Example:2

Using truth tables, prove the followinglogical equivalences:

i. (p ∧ q) ≡ ~ (p → ~ q)
ii. p ↔ q ≡ (p ∧ q) ∨ (~p ∧ ~q)

Solution: i. (p ∧ q) ≡ ~ (p → ~ q)

p	q	p ∧ q	~q	p → ~q	~ (p → ~q)
T	T	T	F	F	T
T	F	F	T	T	F
F	T	F	F	T	F
F	F	F	T	T	F

The entries in the columns 3 and 6 are identical.

∴ (p ∧ q) ≡ ~ (p → ~q)

ii. p ↔ q ≡ (p ∧ q) ∨ (~p ∧ ~q)

P	q	p ↔ q	~p	~q	p ∧ q	~p ∧ ~q	(p ∧ q) v (~p ∧ ~q)
T	T	T	F	F	T	F	T
T	F	F	F	T	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	T	F	T	T

The entries in columns 3 and 8 are identical.

∴ p ↔ q ≡ (p ∧ q) ∨ (~p ∧ ~q)

Example:3

Using truth tables examine whether the following statement patterns are tautology, contradiction or contingency.

i. $(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$

ii. $[(p \vee q) \vee r] \leftrightarrow [p \vee (q \vee r)]$

iii. $(p \wedge q) \vee (p \wedge r)$

Solution: i.

P	q	~q	$p \wedge \sim q$	$p \rightarrow q$	$(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	F	T	F

In the above truth table, all the entries in the lastcolumn are F.

∴ $(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$ is a contradiction.

ii.

P	q	r	$p \vee q$	$q \vee r$	$(p \vee q) \vee r$	$p \vee (q \vee r)$	$[(p \vee q) \vee r] \leftrightarrow [p \vee (q \vee r)]$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	F	F	F	T

In the above truth table, all the entries in the lastcolumn are T.

∴ $[(p \vee q) \vee r] \leftrightarrow [p \vee (q \vee r)]$ is a tautology

iii.

p	q	r	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T

T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	F	T	F
F	F	F	F	F	F

In the above truth table, the entries in the lastcolumn are a combination of T and F.

∴ (p v q) Λ (p v r) is a contingency.

Arguments:

- A valid argument is a finite set of propositions P1, ..., Pr called premises, together with a proposition q, the conclusion, such that the propositional form (P1 ^ P2 ^ ... ^ Pr) → q is a tautology. We say q follows logically from, or is a logical consequence of the premises.
- We write P1, ... , Pr → q. The symbol ` is called the turnstile.
- If an argument in not valid we say that it is invalid.

Example:4

Test the validity of the arguments:
"If Mary graduates then she gets a job".
"'Mary does not get a job".
"Therefore Mary does not graduate".

Solution:

Let
p: Mary graduates
q: she gets a job
The premises are p→ q, ~q
The conclusion are $\sim p$
The arguments is p→ q, ~q $- \sim p$

In order to thest the validity of the arguments we have to show that ((p→ q)Λ ~q) → $\sim p$ is a tatulogy.

P	q	$\sim p$	~q	(p→ q)	(p→ q)Λ ~q	((p→ q)Λ ~q) → $\sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

∴ The last column all values true. So, the argument is valid.

M.M.E.S WOMENS ARTS AND SCIENCE COLLEGE

(Affiliated To Thiruvalluvar University)

HAKEEM NAGAR- MELVISHARAM -632 509

MATHEMATICAL FOUNDATION – I E-NOTES/ MATHEMATICS

UNIT: SET THEORY

Sets:

- A set is a collection of well-defined objects. The objects in a set are called elements or members of that set.
- We generally use capital letters like A, B, X, etc. to denote a set. We shall use small letters like x, y, etc. to denote elements of a set. We write x ! Y to mean x is an element of the set Y . We write t b Y to mean t is not an element of the set Y .

Examples

i. The set of all high school students in Tamil Nadu.
ii. The set of all positive even integers.

Finite set

- A set is said to be a finite set if it contains only a finite number of elements in it.
- If a set X is finite, then we define the cardinality of X to be the number of elements in X .
- Cardinality of a set X is denoted by n(X) .

Examples

i. The set of all days in a week.
ii. The set of all students in a class

Infinite set

- A set which is not finite is called an infinite set.
- If a set X is infinite, then we denote the cardinality of X by a symbol ∞.

Examples

i. The set of all natural numbers .
ii. The set of all rational numbers

Representing a set:

There are two methods of representing a set

(i) Roster or tabular form

- In roster form, all the elements of a set are listed, the elements are being separated by commas and are enclosed within braces { }.
- For example, the set of all even positive integers less than 7 is described in roster form as {2, 4, 6}.

(ii) Set builder

- In set-builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set.
- For example, in the set {a, e, i, o, u}, all the elements possess a common property, namely, each of them is a vowel in the English alphabet, and no other letter possess this property. Denoting this set by V,
- we write V = {x : x is a vowel in English alphabet}

Singleton set:

- If a set A has only one element, we call it a singleton set. Thus,{ a } is a singleton set.

Null set:

- A set which does not contain any element is called the empty set or the null set or the void set.

Example:

i. The set of all even primes greater than 9.

Subset

- A set A is said to be a subset of a set B if every element of A is also an element of B.
- In other words, A ⊂ B if whenever *a* ∈ A, then *a* ∈ B. It is often convenient to use the symbol“⇒” which means *implies*. Using this symbol, we can write the definition of *subset* as follows:
- $A \subset B$ if $a \in A \Rightarrow a \in B$

Example:

i. The set of all natural numbers is a subset of the set of all integers.

Set Equality:

- Two sets *A* and *B* are said to be equal if both contain exactly same elements.
- In such a case, we write *A* =*B* . That is, *A* =*B* if and only if A ⊂ B and B⊂ A.

Example:

i. Let A={1,2,3}, B=The set of all natural numbers less than 4 then A=B

Operations on sets

Union of Sets :

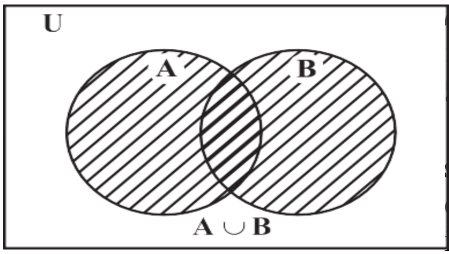
- The union of any two given sets A and B is the set C which consists of all those elements which are either in A or in B. In symbols,
- we write C = A ∪ B = {x | x ∈A or x ∈B}
- Example: Let A = { 2, 4, 6, 8} and B = { 6, 8, 10, 12}. Find A ∪ B.

  We have A ∪ B = { 2, 4, 6, 8, 10, 12} Note that the common elements 6 and 8 have been take only once while writing A ∪ B

Intersection of sets:

- The intersection of two sets A and B is the set which Consists of all those elements which belong to both A and B.
- wewrite A ∩ B = {x : x ∈ A and x ∈ B}.
- Example Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and B = { 2, 3, 5, 7 }. Find A ∩ B

  A ∩ B = { 2, 3, 5, 7 }



Difference of sets:

- The difference of two sets A and B, denoted by A – B is defined as set of elements which belong to A but not to B.
- We write A – B = {x : x ∈ A and x ∉ B} also, B – A = { x : x ∈ B and x ∉A}
- Example: Let A = {1, 2, 3, 4, 5, 6}, B = {2, 4, 6, 8}. Find A – B and B – A.

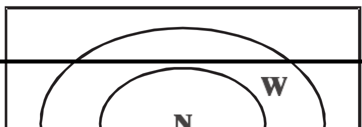
  We have, A – B = {1, 3, 5}, since the elements 1, 3, 5 belong to A but not to B and B – A = {8}, since the element 8 belongs to B and not to A. We note that A – B ≠ B – A.

Complement of a set:

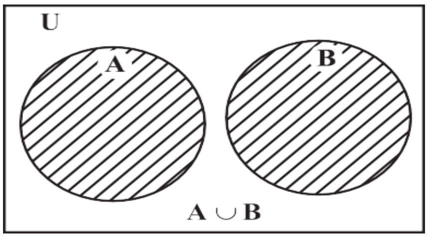
- Let U be the universal set and A a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A.
- We write A′ = {x: x ∈ U and x ∉ A}. Also A′ = U – A
- Example: Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and A = {1, 3, 5, 7, 9}. Find A′. We note that 2, 4, 6, 8, 10 are the only elements of U which do not belong to Hence A′ = { 2, 4, 6, 8,10 }.

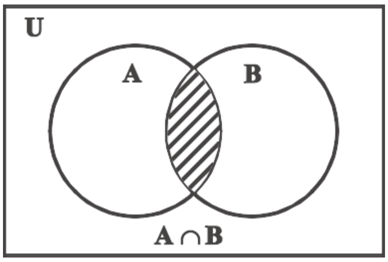
Venn diagrams

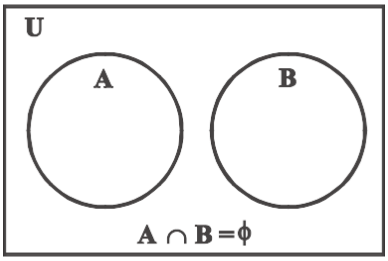
- Venn Diagrams are the diagrams which represent the relationship between sets.
- For example, the set of natural numbers is a subset of set of whole numbers which is a subset of integers.

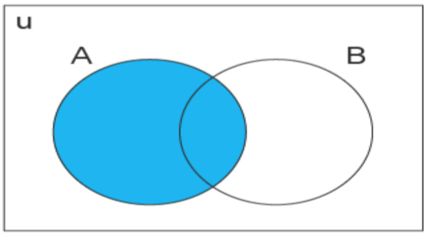


Below we give some Venn diagrams 0f union, intersection, difference and complement of sets.

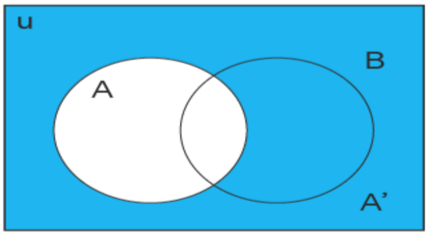




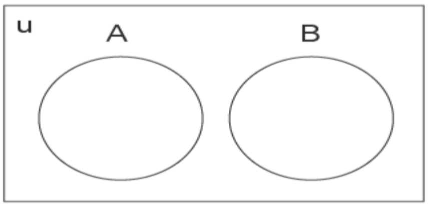




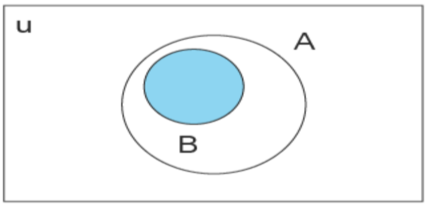
Set A



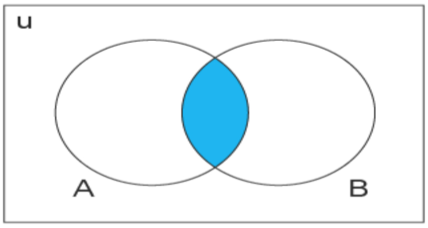
A' is the complement of A



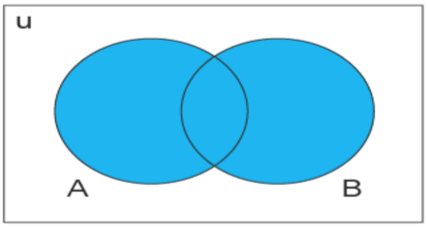
A and B are disjoint sets



$B \subset A$
B is proper subset of A



$A \cap B$



$A \cup B$

Laws of Algebra of Sets

For three sets A, B and C

(i) Commutative Law

a) $A \cap B = B \cap A$

b) $A \cup B = B \cup A$

(ii) Associative Laws

a) $(A \cap B) \cap C = A \cap (B \cap C)$

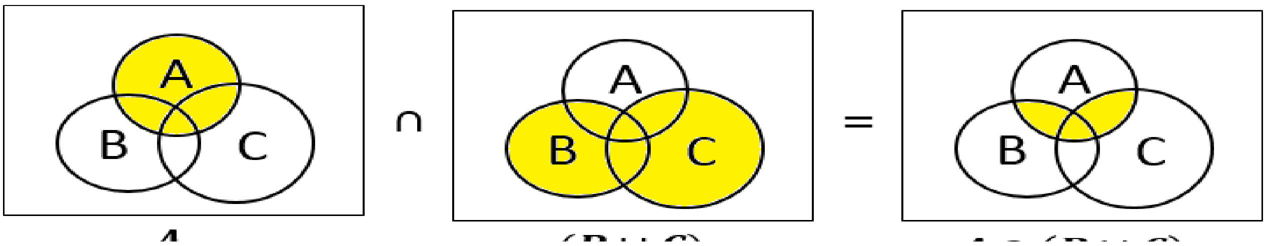
b) (A ∪ B) ∪ C = A ∪ (B ∪ C)

(iii) Distributive Laws

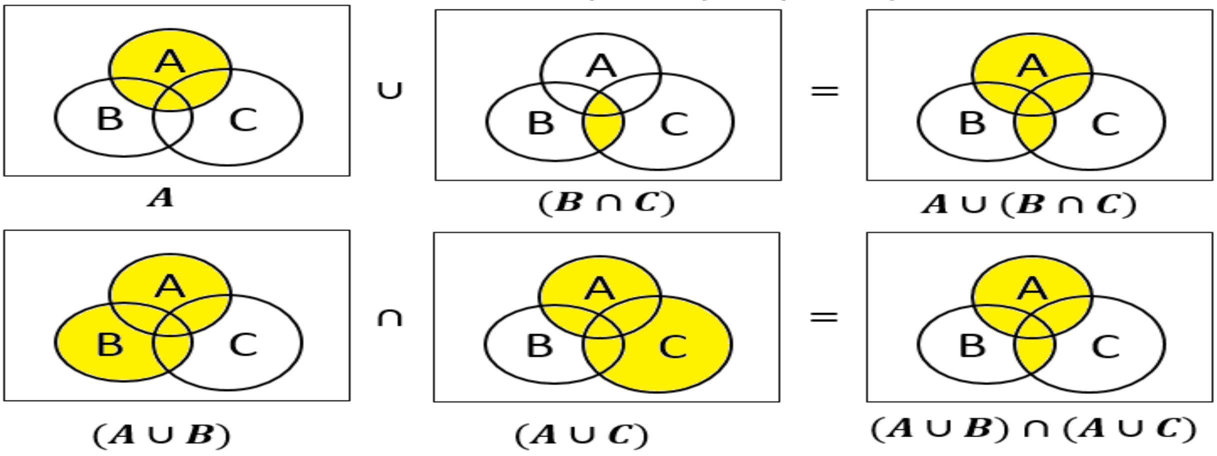
a) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

b) A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$



$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$



(iv) Idempotent Laws

a) A ∩ A = A

b) A ∪ A = A

(v) Identity Laws

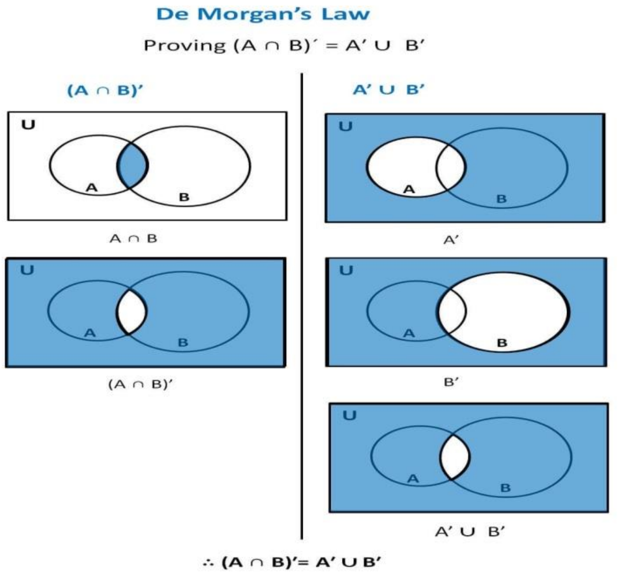
a) A ∪ Φ = A

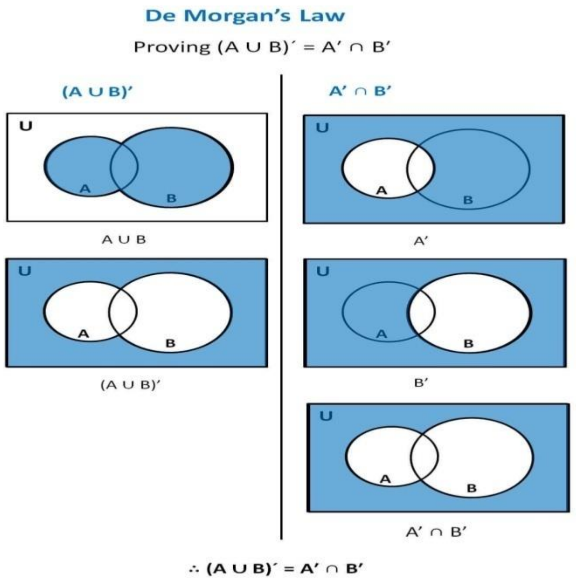
b) A ∩ **U** = A

(vi) De Morgan's Laws

a) (A ∩ B) ′ = A ′ ∪ B ′

b) (A ∪ B) ′ = A ′ ∩ B ′





Example: 1

Given, A = {1, 2, 3, 4, 5}, B = {3, 4, 5, 6} and C = {5, 6, 7, 8}, show that (A U B) U C = A U (B U C)

Solution:

Now, B U C = {3, 4, 5, 6} U {5, 6, 7, 8} = {3, 4, 5, 6, 7, 8}

A U (B U C)= {1, 2, 3, 4, 5} U {3, 4, 5, 6, 7, 8} = {1, 2, 3, 4, 5, 6, 7, 8}.............. (1)

(A U B) = {1, 2, 3, 4, 5} U {3, 4, 5, 6} = {1,2,3,4,5,6}

(A U B) U C = {1,2,3,4,5,6} U {5,6,7,8} = {1, 2, 3, 4, 5, 6, 7, 8}................................. (2)

From (1) and (2), we have (A U B) U C = A U (B U C)

Example: 2

Let A = {0,1,2,3,4}, B = {1, - 2, 3,4,5,6} and C = {2,4,6,7}.

Show that A U (B ∩ C) = (A U B) ∩ (A U C)

Solution:

First, we find A U (B ∩ C)

Consider (B ∩ C) = {1, - 2, 3, 4, 5, 6} ∩ {2, 4, 6, 7} = {4, 6}

A U (B ∩ C) = {0,1, 2, 3, 4} U {4, 6}= {0,1,2,3,4,6}....................... (1)

Next, consider

A U *B* = {0,1,2,3,4} U {1, - 2, 3,4,5,6}= {- 2, 0,1, 2, 3, 4, 5, 6},

A ∪ C = {0,1,2,3,4} ∪ {2,4,6,7} = {0,1, 2, 3, 4, 6, 7}.

Thus
(A ∪ B) ∩ (A ∪ C) = {- 2, 0,1, 2, 3, 4, 5, 6} ∩ {0,1, 2, 3, 4, 6, 7}
(A ∪ B) ∩ (A ∪ C) = {0,1, 2, 3, 4, 6}.. (2)
From (1) and (2) ,we get A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

Example: 3

Let U ={ -2, - 1, 0,1, 2, 3,4,5,6,7,8,9,10}, A = {- 2, 2,3,4,5} and B = {1,3,5,8,9}.
Verify (A ∪ B) ′ = A ′ ∩ B ′
Solution:
(A ∪ B)= {- 2, 2,3,4,5} ∪ {1,3,5,8,9} = {- 2,1, 2, 3, 4, 5, 8, 9}

(A ∪ B) ′ = {- 1, 0, 6, 7, 10}. .... (1)

now, we find A ′ = {- 1, 0,1, 6,7,8,9,10}
B ′= {- 2,- 1, 0, 2,4,6,7,10}.
A ′ ∩ B ′ = {- 1, 0,1, 6,7,8,9,10}∩ {- 2,- 1, 0, 2,4,6,7,10}.
A ′ ∩ B ′ = {- 1, 0, 6, 7,10}...(2)

From (1) and (2) (A ∪ B) ′ = A ′ ∩ B ′

Ordered Pair
- An ordered pair consists of two objects or elements in a given fixed order.

Equality of Ordered Pairs :
- Two ordered pairs (a1, b1) and (a2, b2) are equal iff a1 = a2 and b1 = b2.

Cartesian product of Sets:
- For two sets A and B (non-empty sets), the set of all ordered pairs (a, b) such that a ∈ A and b ∈ B is called Cartesian product of the sets A and’ B, denoted by AxB.
- A x B= {(a,b):a ∈ A and b ∈ B}
- If there are three sets A, B, C and a ∈ A, be B and c ∈ C, then we form, an ordered triplet (a, b,c). The set of all ordered triplets (a, b, c) is called the Cartesian product of these sets A, B and C.
- i.e., A x B x C = {(a,b,c):a ∈ A,b ∈ B,c ∈ C}

Properties of Cartesian Product :

1. A x (B ∪ C)= (A x B) ∪ (A x C)
2. A x (B ∩ C) = (A x B) ∩ (A x C)

Example: 3

Let A = {1, 2, 3} and B = {a, b}. Write *A* x *B* and *B* x *A*?

Solution:

$A \times B = \{1,2,3\} \times \{a,b\} = \{(1,a),(1,b),(2,a),(2,b),(3,a),(3,b)\}$

$B \times A = \{a,b\} \times \{1,2,3\} = \{(a,1), (a,2), (a,3),(b,1), (b,2), (b,3)\}$

Relation

- If A and B are two non-empty sets, then a relation R from A to B is a subset of A x B.
- If R ⊆ A x B and (a, b) ∈ R, then we say that a is related to b by the relation R, written as aRb.

Domain and Range of a Relation

- Let R be a relation from a set A to set B. Then, set of all first components or coordinates of the ordered pairs belonging to R is called : the domain of R, while the set of all second components or coordinates = of the ordered pairs belonging to R is called the range of R.
- Thus, domain of R = {a : (a , b) ∈ R} and range of R = {b : (a, b) ∈ R}

Inverse Relation

- If A and B are two non-empty sets and R be a relation from A to B, such that R = {(a, b) : a ∈A, b ∈ B}
- The inverse of R is denoted by R^{-1} , a relation from B to A and is defined by R^{-1} = {(b, a) : (a, b) ∈ R}

Equivalence Relation

- A relation R is said to be an equivalence relation, if it is simultaneously reflexive, symmetric and transitive on A.

Reflexive Relation

- A relation R is said to be reflexive relation, if every element of A is related to itself.
- Thus, (a, a) ∈ R, ∀ a ∈ A = R is reflexive.

Symmetric Relation

- A relation R is said to be symmetric relation, iff (a, b) ∈ R (b, a) ∈ R,∀ a, b ∈ A
- i.e., a R b ⇒ b R a,∀ a, b ∈ A R is symmetric.

Anti-Symmetric Relation

- A relation R is said to be anti-symmetric relation. iff (a, b) ∈ R and (b, a) ∈ R ⇒ a = b,∀ a, b∈ A

Transitive Relation

- A relation R is said to be transitive relation, iff (a, b) ∈ R and (b, c)∈ R ⇒ (a, c) ∈ R,

∀ a, b, c ∈ A

Equivalence Classes of an Equivalence Relation

- Let R be equivalence relation in A (≠ Φ).
- Let a ∈ A. Then, the equivalence class of a denoted by [a] or {a} is defined as the set of all those points of A which are related to a under the relation R.

Example:5

Let A = {1,3,5,7} and B = {4,8}. If R is a relation defined by "is less than" from A to B find R and R^{-1}

Solution:

$A \times B$ = {(1,4), (1,8), (3,4), (3,8), (5,4), (5,8), (7,4),(7,8)}

Then, 1R4 (1 is less than 4). Similarly, it is observed that 1R8, 3R4, 3R8, 5R8, 7R8

Equivalently R= {(1,4), (1,8), (3,4), (3,8), (5,8), (7,8)}

R^{-1} ={(4,1), (8,1), (4,3), (8,3), (8,5), (8,7)}

Functions:

- A function is defined as a relation **f** from A to B (where A and B are two non-empty sets) such that for every a ∈ A, there is a unique element b∈ B such that (a, b) ∈ f. 'a' is called pre-image and 'b' is called image of function f.
- Hence if f: A → B is a function, then for each element of set A, there is a unique element in set B.
- **Domain and co-domain** – if f is a function from set A to set B, then A is called Domain and B is called co-domain.
- **Range** – Range of f is the set of all images of elements of A. Basically Range is subset of co- domain.
- **Image and Pre-Image** – b is the image of a and a is the pre-image of b if f(a) = b.

Types of functions:

One to one Function(or Injective) :

- A function f is injective if and only if for all a and b in A, if $f a=(b)$, then $a=b$; that is, $f a=f(b)$ implies $a=b$. Equivalently, if $a\neq b$, then $(a)\neq f(b)$.

Onto function / Surjective

- A function $f : A\to B$ is said to be onto function if the range of f is equal to the co-domain of f.

Into function:

- A function $f : A \rightarrow B$ is called an into function if there exists atleast one element in B which is not the image of any element of A.

One–one and onto function:

- If a function $f : A \rightarrow B$ is both one–one and onto, then f is called a bijection from A to B.

Many – one function:

- A function $f: A \rightarrow B$ is called many-one function if two or more elements of A have same image in B.

Composition of Functions:

- Let $f : A \rightarrow B$ and $g\ B \rightarrow C$ be two functions Then the composition of f and g denoted by $g \circ f$ is defined as the function $(g \circ f)\ x = g(f(x))$, $x \epsilon A$

- Composition of three functions is always associative. That is, $f \circ (g \circ h) = (f \circ g) \circ h$