DIFFERENTIAL EQUATIONS

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Syllabus

Objectives

This course aims to provide logical skills in the formation of differential equations, to expose to different techniques of finding solutions to these equations and in addition stress is laid on the application of these equations in geometrical and physical problems.

UNIT-I Ordinary Linear Differential Equations

Bernoulli Equation - Exact Differential Equations - Equations Reducible to Exact Equations - Equations of First order and Higher degree: Equations solvable for p, Equation solvable for x and Equations Solvable for y - Clairaut's Equation.

UNIT-II Ordinary Linear Differential Equations [Contd...]

Method of Variation of Parameters - 2 nd order Differential Equations with Constant Coefficients for finding the P.I's of the form eax V, where V is sin(mx) or cos(mx) and xn - Equations reducible to Linear equations with constant coefficients - Cauchy's homogeneous Linear Equations - Legendre's Linear Equations.

UNIT-III Differential Equations of Other Types

Simultaneous Equations with Constant coefficients - Total Differential Equations Simultaneous Total Differential Equations - Equations of the form dx/P = dy/Q = dz/R

UNIT-IV Laplace Transform

Transform-Inverse Transform - Properties - Application of Laplace Transform to solution of first and second order Linear Differential equations [with constant coefficients].

UNIT-V Partial Differential Equations

Formation of PDF - Complete Integral - Particular Integral - Singular Integral - equations Solvable by direct Integration - Linear Equations of the first order - Non-linear Equations of the first Order: **Types:** $f[p, q] = 0$, $f[x, p, q] = 0, f[y, p, q] = 0, f(z, p, q] = 0, f[x, q] = f[y, p]$, $z = px + qy + f[p, q]$

Recommended Text

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List of Abbreviations

Abbreviation	Description
DE	Differential equation
CF	Complementary function
PI	Particular integral

List of Notations and Symbols

Notation	**Description**
$\mathbb{N}$	The set of natural numbers
$\mathbb{R}^+$	The set of positive real numbers
$\mathbb{R}$	The set of real numbers
$\mathbb{R}^n$	The n-dimensional Euclidean space
$\mathbb{R}^{m\times n}$	The set of $m \times n$ real matrices

Chapter 1

Ordinary Linear Differential Equations

1.1 FIRST ORDER AND FIRST DEGREE EQUATIONS OF BERNOULLI:

The standard form of first order linear differential equations is

$$\frac{dy}{dx} + P(x)y = Q(x)$$

and the general solution for the above equation is

$$ye^{\int Pdx} = \int Qe^{\int Pdx}dx + C$$

since, $e^{\int Pdx}$ is an integral part.

BERNOULLI'S EQUATION: A first order differential equation can be written in the form:

$$\frac{dy}{dx} + P(x)y = Q(x)y^n,$$

where, $P(x)$ and $Q(x)$ are functions of x only (free from y) is called a Bernoulli's equation.

PROBLEMS:

1. Solve $\frac{dy}{dx} + y\tan x = \cos x$

Soln: $\frac{dy}{dx} + y\tan x = \cos x$

This is a standard linear equation in y.

Here, $P = \tan x,\ Q = \cos x$.

The general solution of the first order DE,

$$ye^{\int P dx} = \int Q e^{\int P dx} dx + C \tag{1.1}$$

Now, $e^{\int P dx} = \sec x$

$\therefore$ eqn.(1.1), becomes,

$$\begin{aligned} y(\sec x) &= \int \cos x \sec x dx + C \\ &= \int dx + C \\ y(\sec x) &= x + C \end{aligned}$$

$\therefore$ The general solution is $y(\sec x) = x + C$.

2. Solve $\frac{1}{x}\frac{dy}{dx} + \frac{y}{x}\tan x = \cos x$

Soln: $\frac{1}{x}\frac{dy}{dx} + \frac{y}{x}\tan x = \cos x$

Multiply by x on both sides,

$\frac{dy}{dx} + y \tan x = x \cos x$

This is a standard linear equation in y.

Here, $P = \tan x,\ Q = x \cos x$.

The general solution of the first order DE,

$$ye^{\int Pdx} = \int Qe^{\int Pdx} dx + C \tag{1.2}$$

Now, $e^{\int Pdx} = \sec x$

$\therefore$ eqn.(1.2), becomes,

$$\begin{aligned} y(\sec x) &= \int x \cos x \sec x dx + C \\ &= \int x dx + C \\ y(\sec x) &= \frac{x^2}{2} + C \end{aligned}$$

$\therefore$ The general solution is $y(\sec x) = \frac{x^2}{2} + C$.

3. Solve $(1 - x^2)\frac{dy}{dx} + 2xy = x\sqrt{1 - x^2}$, given that $y = 0, when, x = 0$.

Soln: $(1 - x^2)\frac{dy}{dx} + 2xy = x\sqrt{1 - x^2}$

Multiply by $(1 - x^2)$ on both sides,

$\frac{dy}{dx} + \frac{2x}{1-x^2} y = \frac{x\sqrt{1-x^2}}{(1-x^2)}$

This is a standard linear equation in y.

Here, $P = \frac{2x}{1-x^2},\ Q = \frac{x\sqrt{1-x^2}}{(1-x^2)}$.

The general solution of the first order DE,

$$ye^{\int Pdx} = \int Qe^{\int Pdx}dx + C \tag{1.3}$$

Now, $\int Pdx = \frac{2x}{1-x^2}dx$

put $t = 1 - x^2, -dt = 2xdx$

$\therefore e^{\int Pdx} = \frac{1}{1-x^2}$

Sub. in eqn. (1.3), we get,

$$\begin{aligned} y\left(\frac{1}{1-x^2}\right) &= \int \frac{x\sqrt{1-x^2}}{1-x^2}\frac{1}{1-x^2}dx + C \\ &= \frac{1}{2}\int \frac{1}{t^{\frac{3}{2}}}2xdx + C, \qquad (\because t = 1 - x^2, -dt = 2x) \\ &= t^{-1/2} + C \\ &= (1-x^2)^{-1/2} + C \\ y\left(\frac{1}{1-x^2}\right) &= \frac{1}{\sqrt{1-x^2}} + C \end{aligned} \tag{1.4}$$

put $x = 0, y = 0$ in the above eqn. we get, $C = -1$,

sub $C = -1$ in (1.4),

$y\left(\frac{1}{1-x^2}\right) = \frac{1}{\sqrt{1-x^2}} - 1$

The general solution is $y = \sqrt{1-x^2} - (1-x^2)$

1.2 EXACT EQUATION:

The differential form $M(x,y)dx + N(x,y)dy$ is said to be exact if there exists a function $f(x,y)$ such that,

$\frac{\partial F}{\partial x} = M$ and $\frac{\partial F}{\partial y} = N$

The total differential of $F(x, y)$ satisfies the relation.

$\partial F(x, y) = M(x, y)dx + N(x, y)dy$

If $M(x, y)dx + N(x, y)dy$ is an exact differential form then $M(x, y)dx + N(x, y)dy = 0$ is called an exact equation.

Theorem: If the first order partial derivatives of $M(x, y)$ and $N(x, y)$ are continuous, then, $Mdx + Ndy = 0$ is an exact eqn iff $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

PROCEDURE FOR SOLVING THE EXACT EQUATION:

- Integrate M with respect to x, keeping y constant.
- Integrate those terms in N not containing x with respect to y.
- The sum of these two integrals equated to C is the solution.

PROBLEMS:

1. Solve $(x^2 - 2xy + 3y^2)dx + (y^2 + 6xy - x^2)dy = 0$

Soln: This is in the form $Mdx + Ndy = 0$

$M = x^2 - 2xy + 3y^2$

$N = y^2 + 6xy - x^2$

Condition for exact equation is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial M}{\partial y} = -2x + 6y$

$\frac{\partial N}{\partial x} = 6y - 2x$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\therefore$ The given eqn. is an exact eqn.

Now integrating M with respect to x and N with respect to y, we get,

$\int Mdx = \frac{x^3}{3} - x^2y + 3xy^2$ and $\int Ndy = \frac{y^3}{3}$

Adding above eqns. we get,

$\int Mdx + \int Ndy = C, x^3 - 3x^2 + 9xy^2 + y^3 = C.$

2. Solve $(1 + e^x y + xe^x y)dx + (xe^x + 2)dy = 0$

Soln: This is in the form $Mdx + Ndy = 0$

$M = 1 + e^x y + xe^x y$

$N = xe^x + 2$

Condition for exact equation is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial M}{\partial y} = e^x + xe^x$

$\frac{\partial N}{\partial x} = xe^x + e^x$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\therefore$ The given eqn. is an exact eqn.

Now integrating M with respect to x and N with respect to y, we get,

$\int Mdx = x + xe^x y$ and $\int Ndy = 2y$

Adding above eqns. we get,

$\int Mdx + \int Ndy = C, x + xe^x y + 2y = C.$

1.3 EQUATIONS REDUCIBLE TO EXACT FORM:

An DE which is not exact, can sometimes be converted into an exact equation by multiplying by an integrating factor. In some cases the integrating factor can be formed by inception eventhough an eqn. is of the $Mdx + Ndy = 0$. There is no general method to obtain the integrating factor. But, here below we suggest some of the standard result which might help to determine the integrating factor.

INCEPTION METHOD

$$1. xdy + ydx = d(xy)$$
$$2. \frac{xdy - ydx}{x^2} = d(\frac{y}{x})$$
$$3. \frac{ydx - xdy}{y^2} = d(\frac{x}{y})$$
$$4. \frac{xdy - ydx}{xy} = dlog(\frac{y}{x})$$
$$5. \frac{xdy - ydx}{x^2 - y^2} = d\left(\frac{1}{2}log\left(\frac{x+y}{x-y}\right)\right)$$
$$6. \frac{xdx - ydy}{x^2 + y^2} = d\left(\frac{1}{2}log(x^2 + y^2)\right)$$

PROBLEM

1. Solve $xdx + ydy = a(x^2 + y^2)dy$

Soln: $\frac{xdx+ydy}{x^2+y^2} = ady$

we know that, $\frac{xdx-ydy}{x^2+y^2} = d\left(\frac{1}{2}log(x^2 + y^2)\right)$

$d\left(\frac{1}{2}log(x^2 + y^2)\right) = ady$

Integrating, on both sides, we get,

$\frac{1}{2}log(x^2 + y^2) = ay + C,$

$log(x^2 + y^2) = 2ay + C$

1.4 DE OF FIRST ORDER AND HIGHER DEGREE

Consider the method of solving the first order differential equation of higher degree since $\frac{dy}{dx}$ is denoted by p that is., $p = \frac{dy}{dx}$.

Such a differential equation can be expressed in the form of $f(x, y, p) = 0$. Here, we consider 3 cases.

1. Equation solvable for p

2. Equation solvable for x

3. Equation solvable for y.

CASE 1. Equation solvable for p

The general eqn of first order and n^{th} degree is of the form

$$\left(\frac{dy}{dx}\right)^n + p_1\left(\frac{dy}{dx}\right)^{n-1} + ... + p_{n-1}\left(\frac{dy}{dx}\right) + p_n = 0$$

put $\frac{dy}{dx} = p$

$$p^n + p_1 p^{n-1} + ... + p_{n-1}p + p_n = 0$$

where, $p_1, p_2, ..., p_n$ are functions of x and y. If it is possible to factorize the LHS into linear factors, the above eqn can be expressed in the form,

$$[p - f_1(x,y)][p - f_2(x,y)]....[p - f_n(x,y)] = 0$$

Equating, each terms to zero and integrating we get,

$[F_1(x,y,c_1)][F_2(x,y,c_2)]......[F_n(x,y,c_n)] = 0$, which is the general solution of p where, $C_1, C_2, ..., C_n$ are arbitrary constant.

CASE 2. Equation solvable for x

STEPS

1. Let $F(x,y,p) = 0 \rightarrow (1)$ be a DE of first order and higher degree.

2. Solvable for independent variable x then we can express in the form $x = g(y,p)$

3. Differentiating above eqn. with respect to y

$$\frac{dx}{dy} = h(y, p, \frac{dp}{dy})$$

4. $\frac{dy}{dx} = p, \ \frac{dx}{dy} = \frac{1}{p}$

$$\frac{1}{p} = h(y, p, \frac{dp}{dy})$$

5. $\Phi(y,p,C) = 0, p = f(y,c)$

6. Eliminating p from (1). substitute $p = f(y,c)$ in (1), then we get the general solution

of the given eqn.

CASE 3. Equation solvable for y

STEPS

1. Let $f(x, y, p) = 0 \to (1)$ be the differential eqn. of first order and higher degree.

$p^2 + px^3 - 2x^2y = 0$

2. If it is solvable for the dependent variable y. $y = g(x, p) \to (2)$

3. Differentiate above eqn. with respect to x

$\frac{dy}{dx} = h(x, p, \frac{dp}{dx}),\ \frac{dy}{dx} = p$

$p = h(x, p, \frac{dp}{dx})$

4. $\Phi(x, p, C) = 0, p = f(x, c)$

5. Eliminating p from (2). substitute $p = f(y, c)$ in (2), then we get the general solution of the given eqn.

PROBLEMS:

1. Solve $xyp^2 + (x + y)p + 1 = 0$

Soln: $xyp^2 + xp + yp + 1 = 0$

$xp(yp + 1) + 1(yp + 1) = 0$

$(xp + 1)(yp + 1) = 0$

$xp + 1 = 0, \qquad yp + 1 = 0$

$dy = -\frac{1}{x}dx, \qquad ydy = -dx$

integrating,

$y + \log x - C_1 = 0 \quad \frac{y^2}{2} + x - C_2 = 0$

$\therefore$ The general soln. of n^{th} degree for p is

$[F_1(x, y, c_1)][F_2(x, y, c_2)]......[F_n(x, y, c_n)] = 0$

$\therefore$ The general soln. of given equation is

$(y + \log x - C_1)(\frac{y^2}{2} + x - C_2) = 0.$

2. Solve $y = 3px + 6y^2p^2$

Soln:$3px = y - 6y^2p^2$

$x = \frac{1}{3}[\frac{y-6y^2p^2}{p}]$

Differentiating with respect to y

$p = -\frac{y}{2}\frac{dp}{dy}$

Integrating,

$\log p = -2\log y + \log c$

$p = \frac{c}{y^2}$

The soln. is $y = 3cx + 6c^2$.

3.Solve$p^2 + px^3 - 2x^2y = 0$

Soln. This is in the form of y

$2x^2y = p^2 + px^3$

$y = \frac{1}{2x^2}(p^2 + px^3)$

Differentiating with respect to x

$\frac{1}{p}dp = \frac{1}{x}dx$

Integrating,

$p = xc \therefore y = \frac{1}{2}(c^2 + cx^2)$ is the general solution.

1.5 CLAIRAUT'S EQUATION:

Differential equation of the form,

$y = px + f(p) \rightarrow (1)$

where $f(p)$ is a function of p alone is called clairauts equation.

Differentiating above equation with respect to x we get, $\frac{dy}{dx} = p(1) + x\frac{dp}{dx} + f'(p)\frac{dp}{dx}$

$\frac{dp}{dx} = 0\ (or)\ x + f'(p) = 0$

Case 1: $\frac{dp}{dx} = 0$

$p = c$

The general solution of given eqn. is $y = cx + f(c)$

Case 2: $x + f'(p) = 0 \rightarrow (2)$

By eliminating p between equations (1) and (2), we get a solution of eqn (1) which is free from any arbitrary constant and this solution can be obtain from eqn (2) by given a particular value for c. This solution is called the singular solution of eqn. (2).

PROBLEMS:

1. Solve $(px - y)(py + x) = 2p$

Soln:$(px - y)(py + x) = 2p \rightarrow (1)\ p^2xy + px^2 - py^2 - xy = 2p$

$Let X = x^2,\ \ Y = y^2$

Differentiating we get,

$P\frac{x}{y} = p \qquad (\because \frac{dy}{dx} = P)$

Sub. in eqn. (1), we get,

$Y = XP - \frac{2P}{P+1}$

This is in the form of clairauts equation, put $P = C$

$Y = XC - \frac{2C}{C+1}$

$y^2 = x^2c - \frac{2c}{c+1}$

Chapter 2

Ordinary Linear Differential Equations (cont...)

2.1 Second order differential equation with constant coefficient:

A second order linear differential equation that can be written in the form

$$a_2(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_0(x)y = b(x) \tag{2.1}$$

Here, $a_2(x),\ a_1(x),\ a_0(x),\ b(x)$ are continuous functions of x on an interval D when a_0, a_1, a_2 are constant we say that the equation has constant coefficient, otherwise it has variable coefficient.

Now we are interested in the linear equation where $a_2(x) \neq 0$ in that case we can write the above equation in the form,

$$\frac{d^2y}{dx^2} + p1(x)\frac{dy}{dx} + q(x)y = g(x) \tag{2.2}$$

where, $p(x) = \frac{a_1(x)}{a_2(x)},\ q(x) = \frac{a_0(x)}{a_2(x)},\ g(x) = \frac{b(x)}{a_2(x)}$

Chapter 3

Differential Equations of other types

3.1 Simultaneous equations with constant coefficients:

PROBLEMS:

1. Solve $2\frac{dx}{dt} + x + \frac{dy}{dt} = \cos t, \quad \frac{dx}{dt} + 2\frac{dy}{dt} + y = 0$

Soln: The given eqn. can be written as,

$(2D + 1)x + Dy = \cos t \rightarrow (1)$

$Dx + (2D + 1)y = 0 \rightarrow (2)$

Solving (1) and (2), we get,

$(3D^2 + 4D + 1)y = \sin t$

Auxillary eqn. $3m^2 + 4m + 1 = 0, \qquad m = -\frac{1}{3}, m = -1$

$C.F = A_1 e^{-\frac{1}{3}t} + B_1 e^{-t}$

P.I= $\frac{1}{3D^2+4D+1} \sin t$

P.I= $-\frac{1}{10}[2\cos t + sint]$

The complete solution is $Y = Ae^{-\frac{1}{3}t} + B_1 e^{-t} + \frac{1}{5}(2\cos t + sint) - \frac{1}{10}(-2\sin t + \cos t)$.

3.2 Total differential equation:

An ordinary differential equation of the first order and the first degree of the form $Pdx + Qdy + Rdz = 0$ is called thetotal differential equation, where P, Q, R are the functions of x, y, z. The conditions for the integrating,

$$P[\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}] + Q[\frac{\partial R}{\partial x} - \frac{\partial p}{\partial z} + R[\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}] = 0$$

If $\mu(x, y, z) = 1$, then the condition for the integrating is given by

$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}, \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}, \frac{\partial R}{\partial x} = \frac{\partial P}{\partial z}$

Thus if this three is satisfied, then the DE is exact.

Procedure for solving the total differential equation in 3 variable

- If the given equation is exact, then the solution is obtained by a regrouping of terms.
- If it is not exact, it may be possible to find an integrating factor.
- If differential equation is homogeneous, one variable say z can separated from the other by the substitution $x = uz,\ y = vz$
- If no integrating factor can be formed, consider one of the variables say x as a constant and integral. As $\phi(z)$ take the total differential of the integral just obtain and compare the coefficient of its differentials with those in the given DE. Thus determine $\phi(z)$.

PROBLEM:

1. Solve $(y + z)dx + (z + x)dy + (x + y)dz = 0$

Soln: This is in the form of $Pdx + Qdy + Rdz = 0$

Here, $P = y + z \qquad Q = z + x \qquad R = x + y$

$\frac{\partial P}{\partial y} = 1 \qquad \frac{\partial Q}{\partial x} = 1 \qquad \frac{\partial R}{\partial x} = 1$

$\frac{\partial P}{\partial z} = 1 \qquad \frac{\partial Q}{\partial z} = 1 \qquad \frac{\partial R}{\partial y} = 1$

Condition for integrability:

$P[\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}] + Q[\frac{\partial R}{\partial x} - \frac{\partial p}{\partial z} + R[\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}] = 0$

$P[1 - 1] + Q[1 - 1] + R[1 - 1] = 0$

$\therefore$ The condition is satisfied for inequality.

$(y + z)dx + (z + x)dy + (x + y)dz = 0$

$ydx + zdx + zdy + xdy + xdz + ydz = 0$

$d(xy) + d(yz) + d(zx) = 0$

Integrating on both sides,

$xy + yz + zx = C$

3.3 Simultaneous total differential equations:

The equation in 3 variables are $Pdx + Qdy + Rdz = 0$ and $P'dx + Q'dy + R'dz = 0$ where, P, Q, R and P', Q', R' are any functions of x, y, z

(i) If each of this equation is integrable and have solutions. $\phi(x, y, z) = C_1$ and $\psi(x, y, z) = C_2$ respectively. Then those equations together the solution of the simultaneous equation.

(ii) Equation is not integrable, then they have to write the following equations,

$\frac{dx}{QR' - Q'R} = \frac{dy}{RP' - R'P} = \frac{dz}{PQ' - P'Q}$.

3.4 Equation of the form $\frac{dx}{p} = \frac{dy}{Q} = \frac{dz}{R}$:

(i) METHOD OF GROUPING If it is possible to take two fraction $\frac{dx}{P} = \frac{dz}{R}$ from which y can be canceled or it is absent leaving equation x and z.

Then integrate, we get $\phi(x, z) = C_1 \rightarrow (1)$

If one variable x is absent (or) can be removed may be with the help of eqn. (1) then $\frac{dy}{Q} = \frac{dz}{R}$.

Then integrate, we get, $\psi(y, z) = C_2$

This two independent solution takes together we get the complete solution of the given total DE.

(i) METHOD OF MULTIPLIERS

$\frac{dx}{p} = \frac{dy}{Q} = \frac{dz}{R} \rightarrow (1)$

Choosing multipliers $l,\ m,\ n$

$\frac{ldx}{lp} = \frac{mdy}{mQ} = \frac{ndz}{nR} = k(say)$

$\frac{ldx+mdy+ndz}{lP+mQ+nR} = k(say).$

Condition:$lP + mQ + nR = 0$

$ldx + mdy + ndz = 0$

Integrating, $\Phi(x, y, z) = C_1$.

Again we choose multipliers $l',\ m',\ n'$

$\frac{l'dx}{l'p} = \frac{m'dy}{m'Q} = \frac{n'dz}{n'R} = k(say)$

$\frac{l'dx+m'dy+n'dz}{l'P+m'Q+n'R} = k(say)$

condition:$l'P + m'Q + n'R = 0$

$l'dx + m'dy + n'dz = 0$

Integrating, $\psi(x, y, z) = C_2$

PROBLEMS:

1. Solve $\frac{dx}{z^2y} = \frac{dy}{z^2x} = \frac{dz}{y^2x}$

Soln: $\frac{dx}{z^2y} = \frac{dy}{z^2x} = \frac{dz}{y^2x}$

$\frac{dx}{z^2y} = \frac{dy}{z^2x}$

$\frac{dy}{z^2x} = \frac{dz}{y^2x}$

$\frac{dx}{z^2y} = \frac{dz}{y^2x}$

From the above equations,

$\frac{dx}{z^2y} = \frac{dy}{z^2x}$

$xdx = ydy$

Integrating, $\frac{x^2}{2} = \frac{y^2}{2} + C_1$

$x^2 - y^2 = C_1$

$\frac{dy}{z^2x} = \frac{dz}{y^2x}$

$y^2dy = z^2dz$

Integrating, $\frac{y^3}{3} = \frac{z^3}{3} + C_2$

$y^3 - z^3 = C_2$

$\therefore$ The complete soln. is $(x^2 - y^2)(y^3 - z^3) = 0$

2. Solve, $\frac{dx}{y-3} = \frac{dy}{3-x} = \frac{dz}{x-y}$

Soln: $\frac{dx}{y-3} = \frac{dy}{3-x} = \frac{dz}{x-y}$

Choosing multipliers, $x, \ y, \ z$

$\frac{xdx}{xy-xz} = \frac{ydy}{yz-xy} = \frac{zdz}{xz-zy} = k$

$xdx + ydy + zdz = 0$

Integrating, $\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = C_1$

$x^2 + y^2 + z^2 = C_1$

Again choosing multipliers 1, 1, 1

$\frac{dx+dy+dz}{y-z+z-x+x-y} = k$

$dx + dy + dz = 0$

Integrating, $x + y + z = C_2$

$\therefore$ The complete soln. is $(x^2 + y^2 + z^2)(x + y + z) = 0$